Section 3-1, Mathematics 108

Quadratic Functions

A quadratic function is a function whose rule is a second degree polynomial.

The form for this type of function is

$$f(x) = ax^2 + bx + c \text{ with } a \neq 0$$

The graph of such a function is always a parabola.

The direction, up or down of the parabola is determined by a.

If $a > 0$ then the parabola has a minimum at its vertex and it goes up.

If $a<0$ then the parabola has a maximum at its vertex and it goes down.

The easiest way to graph a quadratic function is to put it into the **standard** form

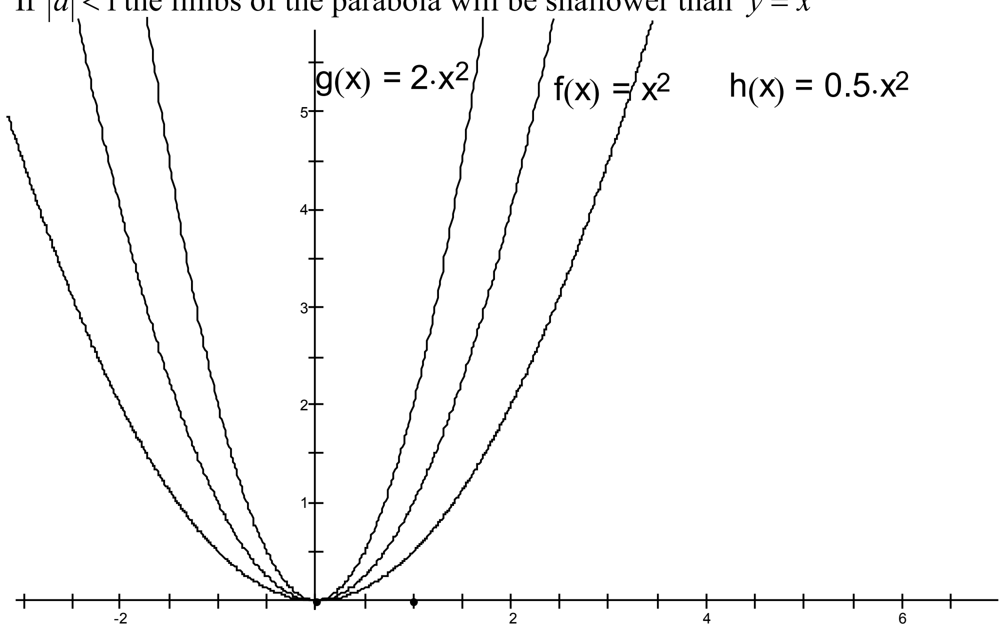
$$y - k = a(x - h)^2$$

In this form we can see immediately that the vertex is at (h,k).

The value of a will control the shape.

If $|a| > 1$ the limbs of the parabola will be steeper than $y = x^2$

If $|a| < 1$ the limbs of the parabola will be shallower than $y = x^2$



Graphing a Quadratic

To put a quadratic into standard form we need to complete the square:

Example:

$$f(x) = 2x^2 - 12x + 13$$

First Factor out *a* value:

$$y = 2\left(x^2 - 6x + \frac{13}{2}\right)$$

The value to complete the square is $\left(\frac{b}{2}\right)^2 = 9$

So we have

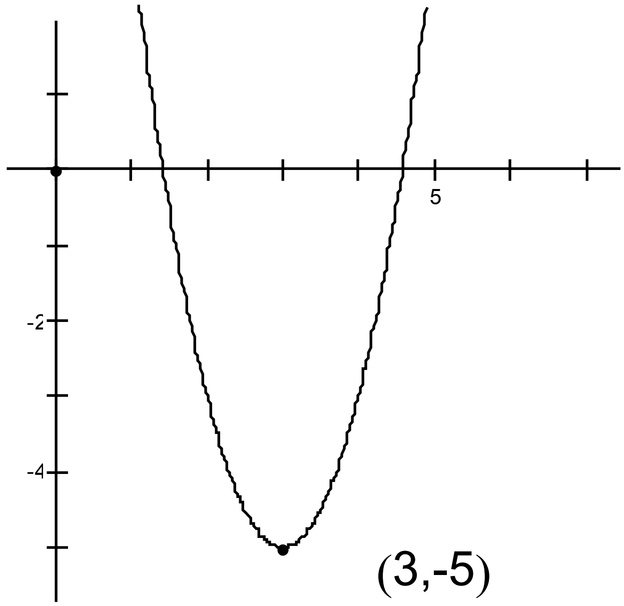
$$y + 18 = 2\left(x^2 - 6x + 9\right) + 13$$
$$y + 18 = 2\left(x - 3\right)^2 + 13$$
$$y + 5 = 2\left(x - 3\right)^2$$
$$y - {}^{-}5 = 2\left(x - 3\right)^2$$

From this we see that the vertex is at (3,-5), the parabola goes up and it goes up twice as fast as x^2

Before we graph this we can see if it has any zeros using the quadratic formula

$$x = \frac{12 \pm \sqrt{144 - 104}}{4} = \frac{12 \pm 2\sqrt{10}}{4} = 3 \pm \frac{\sqrt{10}}{2} \approx 1.4, 4.6$$

which can help in graphing.



Note that he minimum is at 3 and has value -5

Finding the Min/Max Algebraically

We can find the Min/Max more generally as follows:

$$y = ax^2 + bx + c$$

$$y - c = a\left(x^2 + \frac{b}{a}x\right)$$

$$y - c + a\left(\frac{b}{2a}\right)^2 = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right)$$

$$y - c + a\left(\frac{b}{2a}\right)^2 = a\left(x + \frac{b}{2a}\right)^2$$

$$y - \left(\frac{4ac - b^2}{4a}\right) = a\left(x + \frac{b}{2a}\right)^2$$

From this we can see that the min/max will occur at

$$x = \frac{-b}{2a}$$

and the value at the min/max will be

$$\frac{b^2 - 4ac}{4a}$$

It's probably best to just remember $x = \frac{-b}{2a}$ and plug this value into the function to find the value.

Example:

$$f(x) = 5x^2 - 30x + 49$$

$$y - 49 = 5(x^2 - 6x)$$
$$y - 49 + 45 = 5(x^2 - 6x + 9)$$
$$y - 4 = 5(x - 3)^2$$

So the vertex is at (3,4)

Since 5 > 0, this parabola has a minimum value y=4.